WS1

Laplace Transforms Review Sheet

Find the LaPlace transform using the definition $\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt$

1. $f(t) = 1 - t$
2. $f(t) = te^{-4t}$
3. $f(t) = \begin{cases} 1 & 0 \le t < 5 \\ 0 & t \ge 5 \end{cases}$
4. $f(t) = \begin{cases} t & 0 \le t < 1 \\ 0 & t \ge 1 \end{cases}$

Find the LaPlace transform for each function

5. $7\mathcal{U}(t-7)$
6. $t^2\mathcal{U}(t-2)$
7. $36e^{-4t}\mathcal{U}(t-2)$
8. t^3e^t
9. $t\cos 3t$
10. $e^{-5t}\cos 3t$
11. t^2+5
12. $2\sinh 4t$
13. $\int_0^t (t-\tau)\sin\tau\,d\tau$

W52

(14) $\int_0^t \sin\tau \, d\tau$

(15) $\int_0^t \tau e^{t-\tau} \, d\tau$

(16) $\int_0^t u(t-\tau-1)\, u(\tau-2) \, d\tau$

(17) $e^{-3t} * e^{2t}$

(18) $t * \sin 4t$

Find the inverse laplace transforms for the following functions

(19) $\dfrac{1}{(s^2+4)(s+1)}$

(20) $\dfrac{e^{-\pi s}}{s^3+s}$

(21) $\dfrac{s}{(s^2+1)^2}$

(22) $\dfrac{1}{s(s+4)}$

(23) $\dfrac{-10}{s^2-25}$

(24) $\dfrac{3(40-s^5)}{s^6}$

WS 3

(25) $\frac{12}{s^8}$

(26) $\frac{6}{s+2}$

(27) $\frac{-14}{se^{2s}}$

(28) $\frac{6e^{4s}+7}{se^{7s}}$

(29) $\frac{s+6}{s^2+2s+5}$

(30) $\frac{2s^2-7s+20}{s(s^2-2s+10)}$

Solve each initial value problem

(31) $y''+6y'+10y=0$ $\quad y(0)=0$, $y'(0)=1$

(32) $y'+y=f(t)$ $\quad y(0)=0$

$$f(t)=\begin{cases}\sin \pi t & 0 \le t < 1\\ 0 & t \ge 1\end{cases}$$

SKIP
Dirac Delta Function
(covered in 7.5)

(33) $y''-4y'+5y=0$ $\quad y(0)=1$, $y'(0)=0$

(34) $y''-y'=e^t$ $\quad y(0)=0$, $y'(0)=1$

LaPlace transforms Review Sheet answers

1. $\frac{s-1}{s^2}$

2. $\frac{1}{(s+4)^2}$

3. $\frac{1-e^{-5s}}{s}$

4. $\frac{-1}{s}e^{-s} - \frac{1}{s^2}e^{-s} + \frac{1}{s^2}$

5. $\frac{7e^{-7s}}{s}$

6. $\frac{2e^{-2s}(2s^2+2s+1)}{s^2}$

7. $e^{-2s}\left(\frac{1}{s+4}\right)$

8. $6(s-1)^{-4} = \frac{6}{(s-1)^4}$

9. $\frac{s^2-9}{(s^2+9)^2}$

10. $\frac{s+5}{s^2+10s+34}$

11. $2s^{-3} + 5s^{-1} = \frac{2}{s^3} + \frac{5}{s}$

12. $\frac{8}{(s^2-16)}$

13. $\frac{1}{s^2} - \left(\frac{1}{s^2+1}\right)$

14. $\left(\frac{1}{s^2+1}\right)\left(\frac{1}{s}\right)$

15. $\left(\frac{1}{s^2}\right)\left(\frac{1}{s-1}\right)$

16. $\frac{e^{-3s}}{s^2}$

17. $\left(\frac{1}{s+3}\right)\left(\frac{1}{s-2}\right)$

18. $\left(\frac{1}{s^2}\right)\left(\frac{2}{s^2+4}\right)$

19. $\frac{1}{10}\left(2e^{-t} - 2\cos 2t + \sin 2t\right)$

20. $(1+\cos t)\,\mathcal{U}(t-\pi)$

21. $\frac{1}{2}t\sin t$

22. $\frac{-1}{4}e^{-4t} + \frac{1}{4}$

23. $e^{-5t} - e^{5t}$

24. $t^5 - 3$

25. $t^7/420$

26. $6e^{-2t}$

27. $\mathcal{L}^{-1}\left\{\frac{-14}{se^{2s}}\right\} = -14\,\mathcal{U}(t-2)$

28 $6u(t-3)+7u(t-7)$

29 $e^{-t}\cos 2t+\frac{5}{2}e^{-t}\sin 2t$

30 $2-e^{t}\sin 3t$

31 $e^{-3t}\sin t$

32 NOT COVERED

33 $y=e^{2t}(\cos t-2\sin t)$

34 $y=te^{t}$

Laplace transforms Review sheet solutions

(1) $\mathcal{L}\{1-t\} = \int_0^\infty e^{-st}(1-t)\,dt$

parts

	D	I
+	t	e^{-st}
−	1	$-\frac{1}{s}e^{-st}$
+	0	$\frac{1}{s^2}e^{-st}$

$$= \lim_{b\to\infty}\left[\int_0^b e^{-st}\,dt - \int_0^b te^{-st}\,dt\right]$$

$$= \lim_{b\to\infty}\left[\left[-\frac{1}{s}e^{-st}\Big|_0^b\right] + \left[+\frac{1}{s}te^{-st} + \frac{1}{s^2}e^{-st}\Big|_0^b\right]\right]$$

$$= \lim_{b\to\infty}\left[-\frac{1}{s}\underbrace{e^{-sb}}_{0} + \frac{1}{s}e^{0} + \frac{1}{s}\underbrace{be^{-sb}}_{0} + \frac{1}{s^2}\underbrace{e^{-sb}}_{0} - \frac{1}{s^2}\right]$$

let $s>0$ (L'Hospital)

$$= \frac{1}{s} - \frac{1}{s^2} = \boxed{\frac{s-1}{s^2}}$$

(2) $\mathcal{L}\{te^{-4t}\} = \int_0^\infty e^{-st}\cdot te^{-4t}\,dt$

$$= \int_0^\infty t\cdot e^{-(s+4)t}\,dt$$

parts

	D	I
+	t	$e^{-(s+4)t}$
−	1	$\frac{-1}{s+4}e^{-(s+4)t}$
+	0	$\frac{1}{(s+4)^2}e^{-(s+4)t}$

$$= \lim_{b\to\infty}\int_0^b te^{-(s+4)t}\,dt$$

$$= \lim_{b\to\infty} \left[\frac{-1}{s+4} \cdot t \cdot e^{-(s+4)t} - \frac{1}{(s+4)^2} e^{-(s+4)t} \right]\Bigg|_0^b$$

0 (L'Hospital)

$$= \lim_{b\to\infty} \left[\left(\frac{-1}{s+4} \cdot b \cdot e^{-(s+4)b} - \frac{1}{(s+4)^2} e^{-(s+4)b} \right) - \left(\frac{-1}{s+4}(0) e^{-(s+4)0} - \frac{1}{(s+4)^2} e^{-(s+4)(0)} \right) \right]$$

0 0 1

$$= \boxed{\frac{1}{(s+4)^2}}$$

(3) $f(t) = \begin{cases} 1 & 0 \le t < 5 \\ 0 & t \ge 5 \end{cases}$

$$\mathcal{L}\{f(t)\} = \int_0^5 e^{-st}(1)\,dt + \int_5^\infty e^{-st}(0)\,dt$$

$$= -\frac{1}{s} e^{-st} \Big|_0^5 + \int_5^\infty 0\,dt$$

constant
Let $c=0$

$$= -\frac{1}{s} e^{-5s} + \frac{1}{s} e^0$$

$$= -\frac{1}{s} e^{-5s} + \frac{1}{s} = \boxed{\frac{1 - e^{-5s}}{s}}$$

S3

4 $f(t) = \begin{cases} t & 0 \le t \le 1 \\ 0 & t \ge 1 \end{cases}$

constant

Let $c = 0$

$$\mathcal{L}\{f(t)\} = \int_0^1 e^{-st}(t)\,dt + \int_1^\infty e^{-st}(0)\,dt$$

parts

	D	I
$+$	t	e^{-st}
$-$	1	$-\frac{1}{s}e^{-st}$
$+$	0	$\frac{1}{s^2}e^{-st}$

$$= -\frac{1}{s}te^{-st} - \frac{1}{s^2}e^{-st}\Big|_0^1$$

$$= \left(-\frac{1}{s}(1)e^{-s} - \frac{1}{s^2}e^{-s(1)}\right) + \left(\cancel{\frac{1}{s}(0)e^{-s(0)}} + \frac{1}{s^2}e^{0}\right)$$

0

$$= \boxed{-\frac{1}{s}e^{-s} - \frac{1}{s^2}e^{-s} + \frac{1}{s^2}}$$

$a=7$

(5) $\boxed{\mathcal{L}\{7\,\mathcal{U}(t-7)\}} = \boxed{7\cdot\frac{e^{-7s}}{s}}$

$\left(\mathcal{L}\{\mathcal{U}(t-a)\} = \frac{e^{-as}}{s}\right)$

(6) $\boxed{\mathcal{L}\{t^2\,\mathcal{U}(t-2)\}}$

$(t-2)^2 = t^2-4t+4$

$(t-2)^2+4t-4=t^2$

$(t-2)^2+4(t-2)+4 = t^2$

need a match

$\mathcal{L}\{((t-2)^2+4(t-2)+4)\,\mathcal{U}(t-2)\}$ $\boxed{a=2}$

use $\mathcal{L}\{f(t-a)\,\mathcal{U}(t-a)\} = e^{-as}F(s)$

$\mathcal{L}\{t^2+4t+4\} = \frac{2}{s^3} + 4\cdot\frac{1}{s^2} + 4\cdot\frac{1}{s}$

final result $= \boxed{\left(\frac{2}{s^3}+\frac{4}{s^2}+\frac{4}{s}\right)e^{-2s}}$

need match

(7) $\boxed{\mathcal{L}\{36e^{-4t}\,\mathcal{U}(t-2)\}}$

$-4(t-2) = -4t+8$

$-4(t-2)+8 = -4t$

$\mathcal{L}\{36e^{-4(t-2)}e^{8}\,\mathcal{U}(t-2)\}$

$= 36e^{8}\,\mathcal{L}\{e^{-4(t-2)}\,\mathcal{U}(t-2)\}$ $\boxed{a=2}$

$= e^{-2s}\,\mathcal{L}\{e^{-4t}\} = \boxed{e^{-2s}\left(\frac{1}{s+4}\right)}$

8 $\boxed{\mathcal{L}\{t^3 e^t\}}$ shift or 3rd derivative

shift

$$= \mathcal{L}\{t^3 \big|_{s \to s-1}\} = \boxed{\frac{6}{(s-1)^4}}$$

$$\mathcal{L}\{t^3\} = \frac{3!}{s^4} = \frac{6}{s^4}$$

9 $\boxed{\mathcal{L}\{t \cos 3t\}}$ first derivative

$$\mathcal{L}\{\cos 3t\} = \frac{s}{s^2+9}$$

$$\frac{d}{ds}\left(\frac{s}{s^2+9}\right) = \frac{(1)(s^2+9) - (s)(2s)}{(s^2+9)^2} = \frac{-s^2+9}{(s^2+9)^2}$$

$$\mathcal{L}\{t \cos 3t\} = (-1)^1\left(\frac{-s^2+9}{(s^2+9)^2}\right) = \boxed{\frac{s^2-9}{(s^2+9)^2}}$$

10 $\boxed{\mathcal{L}\{e^{-5t} \cos 3t\}}$ shift

$$= \mathcal{L}\{\cos 3t \big|_{s \to s+5}\} = \boxed{\frac{(s+5)}{(s+5)^2+9}}$$

$$\mathcal{L}\{\cos 3t\} = \frac{s}{s^2+9}$$

11 $\boxed{\mathcal{L}\{t^2+5\}} = \mathcal{L}\{t^2\} + \mathcal{L}\{5\}$

$= \boxed{\dfrac{2}{s^3} + \dfrac{5}{s}}$

12 $\boxed{\mathcal{L}\{2\sinh 4t\}} = 2 \cdot \left(\dfrac{4}{s^2-16}\right) = \boxed{\dfrac{8}{(s^2-16)}}$

$k=4$

13 $\boxed{\mathcal{L}\left\{\int_0^t (t-\tau)\sin\tau \, d\tau\right\}}$ convolution!

$= \mathcal{L}\{\sin t * t\} = \boxed{\left(\dfrac{1}{s^2+1}\right)\left(\dfrac{1}{s^2}\right)}$

15 $\boxed{\mathcal{L}\left\{\int_0^t \tau e^{t-\tau} d\tau\right\}}$ convolution!

$f=t \quad g=e^t$

$= \mathcal{L}\{t * e^t\} = \boxed{\left(\dfrac{1}{s^2}\right)\left(\dfrac{1}{s-1}\right)}$

14 $\boxed{\mathcal{L}\left\{\int_0^t \sin\tau \, d\tau\right\}}$ convolution!

$f=\sin t \quad g=1$

$\mathcal{L}\{\sin t * 1\} = \boxed{\left(\dfrac{1}{s^2+1}\right)\left(\dfrac{1}{s}\right)}$

16 $\boxed{\mathcal{L}\left\{\int_0^t u(t-\tau-1)\,u(\tau-2)\,d\tau\right\}}$

$g=(t-\tau)$ piece $\boxed{u(t-1)}$ $\quad f=t$ piece $\boxed{u(t-2)}$ $\quad = \boxed{\frac{e^{-3s}}{s^2}}$

$= \mathcal{L}\{u(t-2) * u(t-1)\} = \left(\frac{e^{-2s}}{s}\right)\left(\frac{e^{-s}}{s}\right)$

17 $\boxed{\mathcal{L}\{e^{-3t} * e^{2t}\}} = \boxed{\left(\frac{1}{(s+3)}\right)\left(\frac{1}{s-2}\right)}$

18 $\boxed{\mathcal{L}\{t * \sin 4t\}} = \boxed{\left(\frac{1}{s^2}\right)\left(\frac{2}{s^2+4}\right)}$

19 $\boxed{\mathcal{L}^{-1}\left\{\frac{1}{(s^2+4)(s+1)}\right\}}$ options: convolution, partial fractions

$\mathcal{L}^{-1}\left\{\frac{A}{s+1} + \frac{Bs+C}{s^2+4}\right\}$

$A(s^2+4) + (Bs+C)(s+1) = 1$

$As^2 + 4A + Bs^2 + Bs + Cs + C = 1$

$s^2(A+B) + s(B+C) + 4A + C = 1$

$A = -B \quad B = -C \quad 4A + C = 1$

so $A = C \quad 4C + C = 1$

$5C = 1$

$\boxed{A = \frac{1}{5}} \quad \boxed{C = \frac{1}{5}} \quad \boxed{B = -\frac{1}{5}}$

$= Ae^{-t} + B\cos 2t + \frac{C}{2}\sin 2t$

$= \boxed{\frac{1}{5}e^{-t} - \frac{1}{5}\cos 2t + \frac{1}{10}\sin 2t}$

21 $\boxed{\mathcal{L}^{-1}\left\{\frac{s}{(s^2+1)^2}\right\}}$

$$= \mathcal{L}^{-1}\left\{\frac{s}{s^2+1} \cdot \frac{1}{s^2+1}\right\}$$ convolution

$$= \cos t * \sin t$$

This was worked out in a previous example. Recall that

$$\int_0^t \sin\tau \cos(t-\tau)\,d\tau$$ (use trig substitutions)

this works out to be $\boxed{\frac{1}{2} t \sin t}$

22 $\boxed{\mathcal{L}^{-1}\left\{\frac{1}{s(s+4)}\right\}}$

convolution or
partial fractions

partial fractions

$$\mathcal{L}^{-1}\left\{\frac{A}{s}+\frac{B}{s+4}\right\} = A\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + B\mathcal{L}^{-1}\left\{\frac{1}{s+4}\right\}$$

$$= \boxed{A + Be^{-4t}}$$

$A(s+4) + B(s) = 1$

$As + 4A + Bs = 1$

$(A+B)s + 4A = 1$

$\boxed{A = \frac{1}{4}}$

$A + B = 0$

$\frac{1}{4} + B = 0$

$\boxed{B = -\frac{1}{4}}$

so $\mathcal{L}^{-1}\left\{\frac{1}{s(s+4)}\right\} = \boxed{\frac{1}{4} - \frac{1}{4}e^{-4t}}$

convolutions

$$\mathcal{L}^{-1}\left\{\frac{1}{s}\cdot\frac{1}{s+4}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s+4}\cdot\frac{1}{s}\right\}$$

$$= e^{-4t} * 1$$

$$= \int_0^t e^{-4\tau}\,d\tau = -\frac{1}{4}e^{-4\tau}\Big|_0^t$$

$$= -\frac{1}{4}e^{-4t} + \frac{1}{4}e^{0} = \boxed{-\frac{1}{4}e^{-4t} + \frac{1}{4}}$$

23 $\boxed{\mathcal{L}^{-1}\left\{\frac{-10}{s^2-25}\right\}} = \mathcal{L}^{-1}\left\{\frac{A}{s+5} + \frac{B}{s-5}\right\}$

$= \boxed{Ae^{-5t} + Be^{5t}}$ now find A and B using partial fractions

$A(s-5) + B(s+5) = -10$
$As - 5A + Bs + 5B = -10$
$s(A+B) + (5B-5A) = -10$

$A+B=0$
$A=-B$
$\boxed{A=1}$

$5B-5A=-10$
$5B-5(-B)=-10$
$10B=-10$
$\boxed{B=-1}$

so $\boxed{e^{-5t} - e^{5t}}$

24 $\boxed{\mathcal{L}^{-1}\left\{\frac{3(40-s^5)}{s^6}\right\}}$

$\mathcal{L}^{-1}\left\{\frac{120}{s^6} - \frac{3s^5}{s^6}\right\} = \mathcal{L}^{-1}\left\{\frac{120}{s^6} - \frac{3}{s}\right\}$

$n=5$ $\quad$ $n=1$

$5! = 5\cdot4\cdot3\cdot2\cdot1 = 120$

$= \mathcal{L}^{-1}\left\{\frac{5!}{s^6}\right\} - 3\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}$

$= \boxed{t^5 - 3}$

25 $\boxed{\mathcal{L}^{-1}\left\{\frac{12}{s^8}\right\}} = \frac{\cancel{4}\cdot\cancel{3}}{7\cdot6\cdot5\cdot\cancel{4}\cdot\cancel{3}\cdot2\cdot1}\mathcal{L}^{-1}\left\{\frac{5040}{s^8}\right\}$

$n=7$
$7! = 7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1$
$= 5040$

$= \boxed{\frac{1}{420}t^7}$

26 $\boxed{\mathcal{L}^{-1}\left\{\frac{6}{s+2}\right\}} = \boxed{6e^{-2t}}$

27 $\boxed{\mathcal{L}^{-1}\left\{\frac{-14}{se^{2s}}\right\}} = \mathcal{L}^{-1}\left\{\frac{-14}{s}e^{-2s}\right\}$

use $\mathcal{L}\{\mathcal{U}(t-a)\} = \frac{e^{-as}}{s}$ with $a=2$

$= \boxed{-14\,\mathcal{U}(t-2)}$

28 $\boxed{\mathcal{L}^{-1}\left\{\frac{6e^{4s}+7}{se^{7s}}\right\}}$

$= \boxed{\mathcal{L}^{-1}\left\{\frac{6e^{4s}}{se^{7s}} + \frac{7}{se^{7s}}\right\}}$

$= \mathcal{L}^{-1}\left\{\underbrace{\frac{6}{s}e^{-3s}}_{a=3} + \underbrace{\frac{7}{s}e^{-7s}}_{a=7}\right\}$

$= \boxed{6\,\mathcal{U}(t-3) + 7\,\mathcal{U}(t-7)}$

29 $\boxed{\mathcal{L}^{-1}\left\{\dfrac{s+6}{s^2+2s+5}\right\}}$

$$\underbrace{s^2+2s+1}+4$$
$$(s+1)^2+4$$

note that this doesn't factor

$$= \mathcal{L}^{-1}\left\{\frac{(s+1)+5}{(s+1)^2+4}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{(s+1)}{(s+1)^2+4}\right\} + \mathcal{L}^{-1}\left\{\frac{5}{(s+1)^2+4}\right\}$$

cos with shift $k=2$ $a=-1$ sin with shift $k=2$ $a=-1$

$$= \boxed{e^{-t}\cos 2t + \frac{5}{2}e^{-t}\sin 2t}$$

30 $\boxed{\mathcal{L}^{-1}\left\{\dfrac{2s^2-7s+20}{s(s^2-2s+10)}\right\}}$

doesn't factor

$2s^2-7s+20$

$(2s- \quad)(s- \quad)$

20·1
10·2
5·4

also doesn't factor

$\boxed{\text{partial fractions}}$

$$\mathcal{L}^{-1}\left\{\frac{A}{s} + \frac{Bs+C}{s^2-2s+10}\right\}$$
$$(s-1)^2+9$$

$$A(s^2-2s+10)+(Bs+C)(s) = 2s^2-7s+20$$
$$As^2-2As+10A + Bs^2+Cs = 2s^2-7s+20$$

$$s^2(A+B)+s(-2A+C)+(10A) = 2s^2-7s+20$$

30 continued

$\boxed{A+B=2}$ $\boxed{-2A+C=-7}$ $\boxed{10A=20}$

$-2(2)+C=-7$ $\boxed{A=2}$

$2+B=2$ $C=-7+4$

$\boxed{B=0}$ $\boxed{C=-3}$

$\boxed{50}$

$$\mathcal{L}^{-1}\left\{\frac{A}{s}+\frac{Bs+C}{(s-1)^2+9}\right\}$$

$$\mathcal{L}^{-1}\left\{\frac{2}{s}+\frac{-3}{(s-1)^2+9}\right\} = \boxed{2-e^t\sin 3t}$$

sin ($K=3$) with shift ($a=1$) $\downarrow$ e^t

31 $\boxed{\text{Solve } y''+6y'+10y=0 \text{ with } y(0)=0,\ y'(0)=1}$

$$\mathcal{L}\{y''\}+6\mathcal{L}\{y'\}+10\mathcal{L}\{y\}=\mathcal{L}\{0\}$$

$$s^2\mathcal{L}\{y\}-s\cancel{y(0)}^{\,0}-\cancel{y'(0)}^{\,1}+6\left[s\mathcal{L}\{y\}-\cancel{y(0)}^{\,0}\right]$$
$$+10\mathcal{L}\{y\}=0$$

$$s^2\mathcal{L}\{y\}-1+6s\mathcal{L}\{y\}+10\mathcal{L}\{y\}=0$$

$$\mathcal{L}\{y\}(s^2+6s+10)=1$$

$$\mathcal{L}\{y\}=\frac{1}{s^2+6s+10}$$

31 $\boxed{\text{continued}}$

$$y = \mathcal{L}^{-1}\left\{\frac{1}{s^2+6s+10}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{(s^2+6s+9)+1}\right\}$$

doesn't factor
$\Rightarrow$ rearrange to sin/cos

$$= \mathcal{L}^{-1}\left\{\frac{1}{(s+3)^2+1}\right\} = \boxed{e^{-3t}\sin t}$$

sin ($k=1$) shift $a=-3$ $\downarrow$ e^{-3t}

EXTRA

32 $\boxed{y' + y = f(t) \quad f(t) = \begin{cases} \sin \pi t & 0 \le t < 1 \\ 0 & t \ge 1 \end{cases} \quad \text{with } y(0) = 0}$

NOT COVERED (Section 7.5)

$\boxed{\text{Dirac Delta function}}$ used for impulse analysis

$$\delta_a(t-t_0) = \begin{cases} 0 & 0 \le t < t_0 - a \\ \frac{1}{2a} & t_0 - a \le t < t_0 + a \\ 0 & t \ge t_0 + a \end{cases}$$

with $\delta(t-t_0) = \lim_{a \to 0} \delta_a(t-t_0)$

and $t_0 > 0$ $\boxed{\mathcal{L}\{\delta(t-t_0)\} = e^{-st_0}}$

33 $\boxed{\text{Solve } y'' - 4y' + 5y = 0 \text{ with } y(0)=1,\ y'(0)=0}$

$\mathcal{L}\{y''\} - 4\mathcal{L}\{y'\} + 5\mathcal{L}\{y\} = \mathcal{L}\{0\}$

$s^2\mathcal{L}\{y\} - s\overset{1}{\cancel{y(0)}} - \overset{0}{\cancel{y'(0)}} - 4[s\mathcal{L}\{y\} - \overset{1}{\cancel{y(0)}}]$
$+ 5\mathcal{L}\{y\} = 0$

$s^2\mathcal{L}\{y\} - s - 4s\mathcal{L}\{y\} + 4 + 5\mathcal{L}\{y\} = 0$

$\mathcal{L}\{y\}(s^2 - 4s + 5) = s - 4$

$\mathcal{L}\{y\} = \dfrac{s-4}{s^2-4s+5} = \dfrac{s-4}{(s^2-4s+4)+1}$

doesn't factor

$y = \mathcal{L}^{-1}\left\{\dfrac{(s-2)}{(s-2)^2+1}\right\} + \mathcal{L}^{-1}\left\{\dfrac{-2}{(s-2)^2+1}\right\}$

cos $k=1$ shift $a=2$ (e^{2t}) sin $k=1$ shift $a=2$ (e^{2t})

$$\boxed{y = e^{2t}\cos t - 2e^{2t}\sin t}$$

34 $\boxed{\text{solve } y'' - y' = e^t \quad \text{with} \quad \begin{matrix} y(0) = 0 \\ y'(0) = 1 \end{matrix}}$

$$\mathcal{L}\{y''\} - \mathcal{L}\{y'\} = \mathcal{L}\{e^t\}$$

$$s^2\mathcal{L}\{y\} - s\underbrace{\cancel{y(0)}}_{0} - \underbrace{\cancel{y'(0)}}_{1} - [s\mathcal{L}\{y\} - \underbrace{\cancel{y(0)}}_{0}] = \frac{1}{s-1}$$

$$s^2\mathcal{L}\{y\} - 1 - s\mathcal{L}\{y\} = \frac{1}{s-1}$$

$$\mathcal{L}\{y\}(s^2 - s) = \frac{1}{s-1} + 1 = \frac{1+s-1}{s-1} = \frac{s}{s-1}$$

$$\mathcal{L}\{y\} = \frac{\cancel{s}}{(s-1)} \cdot \frac{1}{\cancel{s}(s-1)} = \frac{1}{(s-1)^2}$$

$$\mathcal{L}\{y\} = \frac{1}{(s-1)^2}$$

shift
$a=1$
(e^t)

$$y = \mathcal{L}^{-1}\left\{ \frac{1}{s^2} \Big|_{s \to s-1} \right\} = te^t$$

$$\boxed{y = te^t}$$